

RADIATION DAMPING AT A BUBBLE WALL

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The first order phase transition proceeds via nucleation and growth of true vacuum bubbles. When charged particles collide with the bubble they could radiate electromagnetic wave. We show that, due to an energy loss of the particles by the radiation, the damping pressure acting on the bubble wall depends on the velocity of the wall even in a thermal equilibrium state.

There have been many studies on cosmological roles of first-order phase transitions which proceed via nucleation and growth of vacuum bubbles. Studying particle scatterings at a moving bubble wall is important to know the bubble dynamics in a hot plasma. For example, to calculate the velocity of electro-weak bubbles[1, 2] and the CP violating charge transport rate by the wall available for baryogenesis, [3] one should know the reaction force acting on the wall by the scattered particle(*e.g.* quarks, gauge bosons).

When charged particles collide with a bubble wall, they could emit electromagnetic radiation. In this paper we study the effect of energy being carried away by the radiation on the pressure acting on the bubble wall. When particles enter a true vacuum bubble through a wall (say, the electroweak bubble), they could acquire a mass and are decelerated. For example, a fermion ψ may get a mass through the well known Yukawa term $g\bar{\psi}\phi\psi = m\bar{\psi}\psi$, where ϕ is a Higgs field. At this time if the particle is charged electromagnetically, it can radiate electromagnetic wave due to the deceleration. Let us calculate the pressure that scattered particles exert on the bubble wall. For simplicity we assume a linear profile for the bubble wall.(See Fig.1.) and choose a rest frame of the bubble wall. Then, $m(x) = m_0x/d$ ($0 < x < d$) is the position dependent rest mass of the particle at the wall. We also assume that the mean free path of the collision is much shorter than the wall width d , hence WKB approximation is good. The radiation power of an accelerated particle is given by the relativistic version of

Lamor's formula:

$$\frac{dE_{rad}}{dt} = \frac{\alpha}{m^2} \left(\frac{d\vec{k}}{dt} \right)^2, \quad (1)$$

where $\alpha = 2e^2/3c^3 \simeq 0.0611$ in natural units $\hbar = c = k = 1$ and $\vec{k} = (k_x, k_y, k_z)$ is a 3-momentum of the particle. Assuming that the wall is planar and parallel to the $y - z$ plane we can treat the bubble as 1-dimensional one along the x -axis. Energy, momentum and mass of the particle satisfy an usual relation

$$E^2 \equiv m^2(x) + |\vec{k}|^2(x). \quad (2)$$

Let us denote k_x as k from now on. Differentiating the above equation with time t and using $k = Edx/dt$, we get a force

$$\frac{dk}{dt} = -\frac{dm^2}{dx} \frac{1}{2E}, \quad (3)$$

which is a starting point of the pressure calculation[4]. However, if we also consider the energy carried away by the radiation E_{rad} , then the total energy conserved is $E_{tot} \equiv E_{rad} + E$ and the force and ,hence, the pressure should be changed.

From $dE_{tot}/dt = 0$ we obtain

$$\frac{k}{E} \frac{dk}{dt} + \frac{m}{E} \frac{dm}{dt} + \frac{\alpha}{m^2} \left(\frac{dk}{dt} \right)^2 = 0, \quad (4)$$

which has a solution

$$\frac{dk}{dt} = \frac{-m^3}{2\alpha E} \left[1 - \sqrt{1 - \frac{4\alpha E k}{m^4} \frac{dm}{dt}} \right]. \quad (5)$$

Up to $O(\alpha)$ we can expand the root term and get

$$\frac{dk}{dt} \simeq -\frac{dm}{dx} \frac{m}{E} - \frac{2\alpha}{kE} \left(\frac{dm}{dx} \right)^2. \quad (6)$$

Then the total pressure by the collision of the particles in the plasma is given by[4]

$$P = \int_{-\infty}^{\infty} dx \int \frac{d^3k}{(2\pi)^3} \left[-\frac{dk}{dt} f(E(k)) \right], \quad (7)$$

where $f(E) = (\text{Exp}(\beta E) \pm 1)^{-1}$ is a distribution function of fermions and bosons respectively. First, let us briefly review the well-known results without the radiation damping($\alpha = 0$). When the mean velocity of the plasma fluid V (or minus of the bubble wall velocity relative to the fluid) is zero, the first term of the Eq.(6) contributes

$$\begin{aligned} P_1 &= \int_{-\infty}^{\infty} \frac{dm^2(x)}{dx} dx \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E} \frac{1}{e^{\beta E} \pm 1} \\ &= F(m, T) - F(0, T), \end{aligned} \quad (8)$$

where $F(\phi, T)$ is the T dependent part of free energy at a temperature $T = \beta^{-1}$.

When $V \neq 0$ the distribution function is changed to

$$f[\gamma(E - Vk)] = \left(e^{\beta\gamma(E - Vk)} \pm 1 \right)^{-1}, \quad (9)$$

where γ is the Lorentz gamma factor. However, using the fact that the phase factor d^3k/E is Lorentz invariant and changing the integration variable to $k' = \gamma(k - VE)$ and defining $E' \equiv \gamma(E - Vk)$ one can find that the V dependency of P_1 disappears [4]. So one have needed to consider a non-equilibrium deviation of f to calculate the velocity of the wall [5].

Now let us consider the effect of the radiation, the second term of Eq.(6).

When $V = 0$ the term contributes

$$P_2 = 2\alpha \int_{-\infty}^{\infty} \left(\frac{dm(x)}{dx} \right)^2 dx \int \frac{d^3k}{(2\pi)^3} \frac{1}{Ek(e^{\beta E} \pm 1)}, \quad (10)$$

which vanishes, because the integrand is an odd function of k . When $V \neq 0$ one can easily check that due to the $1/k$ term the V dependency survives even under the change of the integration variable. So in this case

$$\begin{aligned} P_2 &= 2\alpha \int_{-\infty}^{\infty} \left(\frac{dm(x)}{dx} \right)^2 dx \int \frac{d^3k}{(2\pi)^3} \frac{1}{Ek(e^{\beta\gamma(E-Vk)} \pm 1)}. \\ &\equiv 2\alpha \int_{-\infty}^{\infty} \left(\frac{dm(x)}{dx} \right)^2 dx I_2(x) \end{aligned} \quad (11)$$

Let us discuss overall features of this integration qualitatively. To investigate a singular behavior of the integrand in Eq.(11) near $k = 0$ we first consider k integration(along x-axis)

$$I \equiv \int_{-\infty}^{\infty} \frac{g(k)}{k} dk = \int_{-\infty}^{\infty} \frac{g(k) - g(0)}{k - 0} dk + g(0) \int_{-\infty}^{\infty} \frac{1}{k} dk, \quad (12)$$

where $g(k) = (E\{Exp[\beta\gamma(E - Vk)] \pm 1\})^{-1}$. The second term of the above equation is zero. For the first term, since the contribution from the integrand with $k \simeq 0$ dominates, one can approximate the term as

$$I \simeq \int_{-k_0}^{k_0} \frac{dg(k)}{dk} dk \simeq g(k_0) - g(-k_0), \quad (13)$$

where $[k_0, -k_0]$ is a small momentum interval in which the integrand contributes significantly. If $V = 0$ two terms cancel each other. When $V \ll 1$ one can expand them up to $O(V)$ and expect that they are linearly dependent on V . On the other

hand, as $V \rightarrow 1$ ($\gamma \rightarrow \infty$) they are exponentially suppressed. The integration over k_y and k_z are just sums of I with a shifted value of E , hence the overall behaviour would not be changed after 3-dimensional calculation. (During the numerical study it is useful to change the measure from $dk_y dk_z$ to $2\pi E dE$) The numerical integration of Eq.(11) confirms this behaviour.(See Fig.2)

To be more concrete, let us obtain an approximate value of the integration when $V \ll 1$ for fermions. In this case we can expand $f[\gamma(E - Vk)] \simeq f(E) - V\beta k f(E)[f(E) - 1] = f(E) + V\beta k f^2(E) \text{Exp}(-\beta E)$. The integration with the first term gives zero and the second term contributes

$$I_2 = V\beta \int \frac{d^3k}{(2\pi)^3} \frac{1}{E} f^2(E) \text{Exp}(-\beta E) = \frac{(\ln 2)TV}{2\pi^2} \quad (14)$$

, because

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{E} f^2(E) \text{Exp}(-\beta E) \simeq \frac{(\ln 2)T^2}{2\pi^2}, \quad (15)$$

to lowest order in $(m/T)^2$ (See Ref.[5]). Therefore, for the wall described in Fig.1, the pressure by the radiation is

$$P_2 = \frac{(\ln 2)\alpha m_0^2 T}{\pi^2 d} V, \quad (16)$$

which well fits with the numerical result shown in Fig.2 when V is small. This pressure is proportional to the wall velocity up to the moderately relativistic case and exists even when the system is in a thermal equilibrium. Since P_2 is proportional to the square of the charge, an antiparticle contributes equally as a particle.

In summary we consider the radiation damping of the particles colliding with bubbles and calculate the velocity dependent pressure on the bubble wall.

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FIG.1.

The mass of the particle $m(x)$ in the wall rest frame.

FIG.2.

The pressure by the radiation damping of fermions colliding with the linear bubble wall (Eq.(11)). Here $1/d = 1 = m_0 = T$ for simplicity.

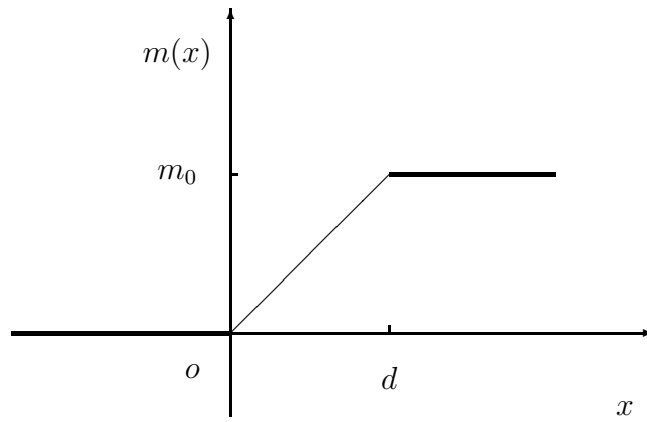


Fig.1.